NUMERICAL COMPUTATION OF THE AVERAGE REYNOLDS STRESS TENSOR IN THE VORTEX CONDENSATE OF 2D TURBULENCE

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In many situations of interest in fluid mechanics, turbulent fluctuations are superimposed on a background mean-flow. The mean-flow may be imposed by an external operator, but it may also be generated spontaneously. The most simple framework for the latter case is probably 2D incompressible flows in a finite-size domain. Indeed, the inverse cascade transfers energy from the injection scale towards larger scale, where it is dissipated by some large-scale friction. When the friction is small enough, the energy accumulates in coherent structures at the box scale [3]. Such structures are often referred to as a *condensate*, since they strongly dominate the energy budget of the system. For instance, in a bi-periodic domain with unit aspect ratio, the condensate takes the form of a pair of vortices of opposite signs.

Due to the time scale separation between the condensate and the incoherent turbulent fluctuations, an asymptotic expansion of the hierarchy of moments can be carried out to obtain closed equations describing both the mean flow and turbulence. These equations provide predictions, for instance, for the mean-flow profile in the vortex condensate [2] (square domain) or in jets at the surface of a sphere [1]. The theoretical computations relate the mean-flow to the turbulent momentum flux across the mean-flow gradient, i.e. the off-diagonal term of the Reynolds stress tensor.

After briefly presenting these analytical predictions, we will test their validity using direct numerical simulations. We shall work in a square domain with periodic boundary conditions, for which a vortex pair forms in the stationnary state (see Fig. 1), and carry out very long (several hundred thousand turnover times) integration of the 2D Navier-Stokes equations with small-scale random forcing and linear friction. In particular, we will discuss how the components of the Reynolds stress tensor scale with both the distance from the vortex core and the large scale friction coefficient, which is the small parameter in the theory.

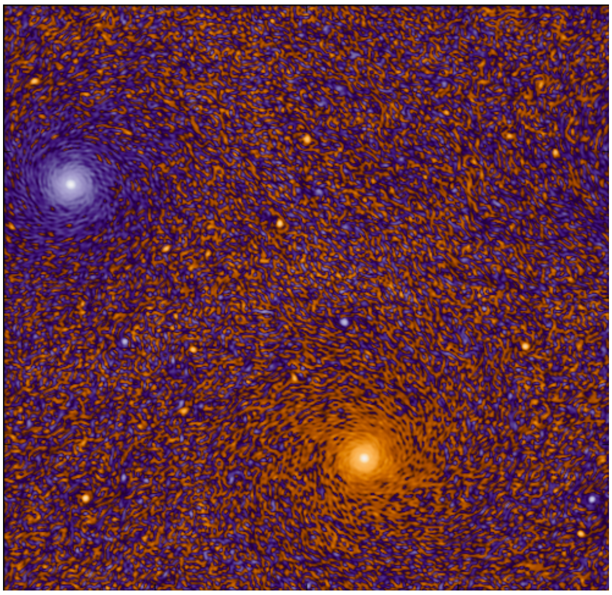

Figure 1. Snapshot of the vorticity field in the condensate state in direct numerical simulations of the incompressible 2D Navier-Stokes equations.

References

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