ENTROPY PRODUCTION IN THE TURBULENT CASCADE – A NEW CRITERION FOR UNIVERSALITY CONSIDERATION

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The analysis of 61 data sets of measurements of developed turbulence in jet flows, and wake flows of a grid and a cylinder are presented. The Taylor Reynolds number covers a range between 100 and 1000. For these data we analyse properties of homogeneous isotropic turbulence. Besides the common scaling analysis of structure functions, $S^n(u_r) =< u_r^n >$, which is determined by the probability density function (pdf) $p(u_r)$, our results are based on a multi-scale analysis, which aims at characterising the joint pdf of the longitudinal velocity increments $u_r(x) = u(x + r) - u(x)$ for different scales $p(u_r; u_{r+\Delta}; ...; u_{r+N\Delta})$. In view of the joint probabilities, we do not split up the cascade process into the statistics of separated increments u_r, as it is done by the structure function analysis, but we analyse the whole cascade process, expressed by the evolution of an increment $u_{(\cdot)}$ from a large scale down to a small scale, typically chosen to be the Taylor length scale. Based on experimental evidence, see [1, 3], we assume that the cascade process is Markovian for a step size Δ larger than the Taylor length scale. This results into a factorisation of the joint pdf

$$p(u_r; u_{r+\Delta}; ...; u_{r+N\Delta}) = p(u_r|u_{r+\Delta})p(u_{r+\Delta}|u_{r+2\Delta}) \times ... \times p(u_{r+(N-1)\Delta}|u_{r+N\Delta})p(u_{r+N\Delta}). \quad (1)$$

Note that this factorisation is a three point closure, as two increments with a common point x are defined by the velocity u at three points. The knowledge of the conditional pdf $p(u_r|u_{r'})$ for $r' > r$ determines now the whole cascade and as shown in [1, 6] can be expressed by a Fokker- Planck equation

$$-\frac{\partial}{\partial r}p(u_r|u_{r'}) = -\frac{\partial}{\partial u_r}\left[D^{(1)}(u_r)p(u_r|u_{r'})\right] + \frac{\partial^2}{\partial u_r^2}\left[D^{(2)}(u_r)p(u_r|u_{r'})\right]. \quad (2)$$

In this stochastic cascade approach one has determine the coefficients $D^{(1)}(u_r)$ and $D^{(2)}(u_r)$, called drift and diffusion coefficient, which give access to the conditional pdfs $p(u_r|u_{r'})$ over the Fokker-Planck equation, and thus also access to the joint pdf of equ. (1). The evolution of a single cascade path $u_{(\cdot)}$ is given by a corresponding Langevin equation, which determines the evolution of u_r in scale by the same $D^{(1)}(u_r)$ and $D^{(2)}(u_r)$. To determine $D^{(1)}(u_r)$ and $D^{(2)}(u_r)$, at first their functional forms are determined by the estimation of the Kramers-Moyal coefficients, which are then successively optimised by solving equ. (2) numerically and minimising the difference between the thus numerically estimated conditional pdf and the experimentally determines ones [4, 2].

Based on this stochastic model of the cascade, i.e. of $u_{(\cdot)}$, it is possible to link the cascade to non-equilibrium thermodynamics. In particular, based on the works [7, 5], the entropy production of each single trajectory $\Delta S[u(\cdot)]$ can be evaluated by

$$\Delta S[u(\cdot)] = \int_L^\lambda \partial_r u_r \, \frac{D^{(1)}(u_r, r) - \partial_u D^{(2)}(u_r, r)}{D^{(2)}(u_r, r)} \, \mathrm{d}r - \ln \frac{p(u_\lambda, \lambda)}{p(u_L, L)}. \quad (3)$$

The knowledge of the entropy production allows to examine the validity of a generalised second law of thermodynamics for turbulent cascades, namely the integral fluctuation theorem (IFT), stating that

$$\left\langle e^{-\Delta S[u(\cdot)]} \right\rangle_{u(\cdot)} = 1. \quad (4)$$

For all 61 data sets we find that the IFT is well fulfilled. i.e. the left side of equ. (4) converges to 1.01 ± 0.01. Hence we take the IFT as a new fundamental law for the turbulent cascade. Based on this conjecture, it can be shown that all cascades process of the measured data exhibit non-scaling contributions. Furthermore, the intermittency causing contribution of the diffusion term $D^{(2)}$ shows clear non-universal behavior, as it depends on the Reynolds number and also, most surprisingly, on the flow type.

References

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